

c-Chart ... 1/2

32

Example 4:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 6, Tue:4, Wed:5, Thu:3, Fri:5. The number of students on roll on these five days remained 40. Develop a control chart.

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is fixed). ∴ Correct Chart is c

Day	Class Strength i.e. Sample Size (n)	Number of Absentees i.e. Defaults/ Defects (d)
Mon	40	6
Tue	40	4
Wed	40	5
Thu	40	3
Fri	40	5

Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$

where

\bar{c} = CL & mean number of defects
 z = Quality standard, eg 3 sigma or more, or degree of confidence

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c-Chart ... 2/2

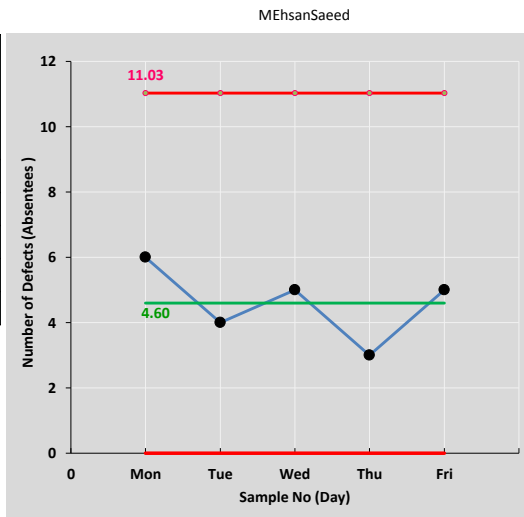
33

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	Cont Limits	
				LCL	UCL
M	40	6		-1.83	11.03
T	40	4		-1.83	11.03
W	40	5		-1.83	11.03
Th	40	3		-1.83	11.03
F	40	5		-1.83	11.03
Σ	200	23			
\bar{c}	23/5 = 4.6 absentees per day/class				

Cont Limits = $\bar{c} \pm z\sqrt{\bar{c}}$

Here, the SD $\sqrt{\bar{c}}$ is constant ∴ LCL & UCL for all observations are also constant, giving straight lines $\bar{c} = \frac{23}{5} = 4.6$, z=3

UCL, LCL = $4.6 + 3\sqrt{4.6} = 11.03 \rightarrow 11$, $-1.83 \rightarrow 0$



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u-Chart ... 1/2

34

Example 5:

A teacher wishes to monitor and control the class attendance. He records the number of absentees over an academic week and the result is Mon: 6, Tue:3, Wed:4, Thu:3, Fri:4. The number of students on roll on these five days were 40, 42, 42, 38, 38 respectively. Develop a control chart for absentees (defects).

Explanation: This is a case of defects (absentees) in a unit (class). The sample size (class strength is variable). ∴ Correct Chart is **u**

Day	Class Strength i.e. Sample Size (n)	Number of Absentees i.e. Defaults/ Defects (d)
Mon	40	6
Tue	42	3
Wed	42	4
Thu	38	3
Fri	38	4

$$\text{Cont Limits} = \bar{u} \pm z \sqrt{\frac{\bar{u}}{n}}$$

where

\bar{u} = CL & mean proportional number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

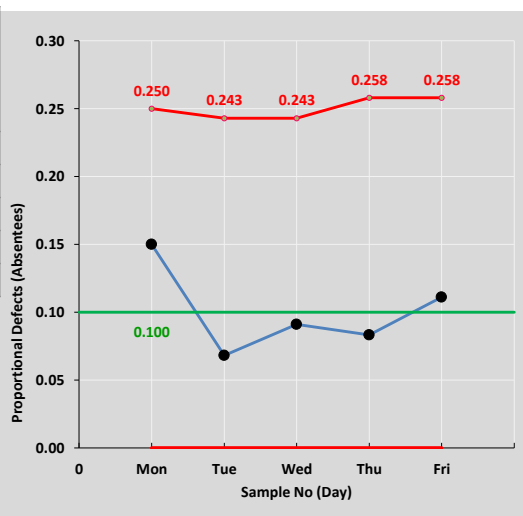
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u-Chart ... 2/2

35

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	LCL	UCL
M	40	6	0.150	-0.050	0.250
T	42	3	0.068	-0.043	0.243
W	42	4	0.091	-0.043	0.243
Th	38	3	0.083	-0.058	0.258
F	38	4	0.111	-0.058	0.258
Σ	200	20	0.100		
\bar{u}	20/200 = 0.10 (10% absentees per day)				

$$\text{Cont Limits} = \bar{u} \pm z \sqrt{\frac{\bar{u}}{n}}$$



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In each case (for each day), the sample size n varies. ∴ the standard deviation $\sqrt{\frac{\bar{u}}{n}}$, and hence **LCL** & **UCL** for each case vary, and form wiggly lines

np-Chart ... 1/2

Example 6: An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. Assuming that the strength of each class is 40, develop a control chart.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is fixed. ∴ Correct Chart is np

$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1 - \bar{p})}$$

where

$$\bar{p} = \text{CL \& mean of sample proportion defectives} = \frac{\text{total defectives}}{\text{total observations}}$$

Z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$$\sigma_{np} = \text{Std Dev of the Average Proportion Defective} = \sqrt{n\bar{p}(1 - \bar{p})}$$

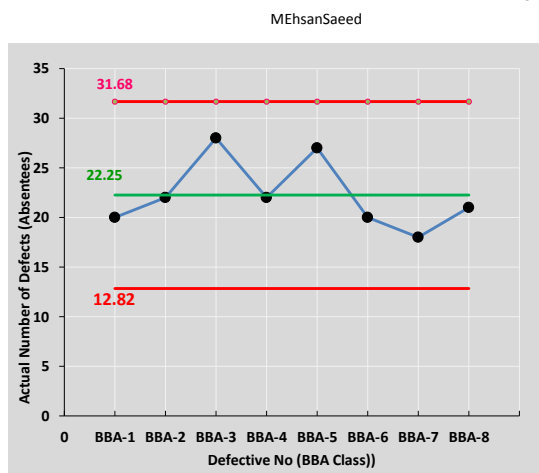
n = mean sample size

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36

np-Chart ... 2/2

Defective BBA-	Sample Size (n)	Defects (D)	(Prop Defects D/n)	LCL	UCL
1	40	20	Not Reqd	12.82	31.68
2	40	22		12.82	31.68
3	40	28		12.82	31.68
4	40	22		12.82	31.68
5	40	27		12.82	31.68
6	40	20		12.82	31.68
7	40	18		12.82	31.68
8	40	21		12.82	31.68
Σ		178			



37

$n\bar{p}$	178/8 = 22.25
\bar{p}	22.25/40 = 0.556

$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1 - \bar{p})}$$

In each case (for each class), sample size **n** is the same. ∴ the SD $\sqrt{n\bar{p}(1 - \bar{p})}$, and hence **LCL & UCL** for each case are also constant, giving straight lines

$$\text{UCL, LCL} = 22.25 \pm 3\sqrt{22.25(1 - 0.556)} = 31.68 \rightarrow 32, 12.82 \rightarrow 13$$

p-Chart ... 1/2

Example 7: An HOD wishes to monitor & control the class attendance in his Dept. He selects the 8 BBA classes and observes the number of absentees over the period of time. The results are BBA-1: 20, BBA-2: 22, BBA-3: 28, BBA-4: 22, BBA-5: 27, BBA-6: 20, BBA-7: 18 & BBA-8: 21. The class strength was 40, 42, 36, 44, 41, 35, 44 & 43 respectively.

Explanation: This is a case of defectives (classes) with defects (absentees). The sample size (class strength) is variable ∴ Correct Chart is **p**.

$$\text{Cont Limits} = \bar{p} \pm z\sigma_p = \bar{p} \pm z \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where

\bar{p} = CL & mean of sample proportion defectives = $\frac{\text{total defectives}}{\text{total observations}}$

z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

σ_p = Std Dev of the Average Proportion Defective = $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

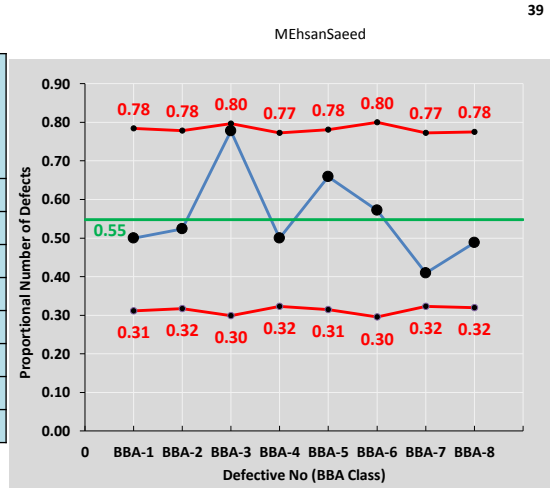
n = mean sample size

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38

p-Chart ... 2/2

Defective (Class) BBA-	Sample Size (n)	Defects (D)	Prop Defects (D/n)	LCL	UCL
1	40	20	0.50	0.31	0.78
2	42	22	0.52	0.32	0.78
3	36	28	0.78	0.30	0.80
4	44	22	0.50	0.32	0.77
5	41	27	0.66	0.31	0.78
6	35	20	0.57	0.30	0.80
7	44	18	0.41	0.32	0.77
8	43	21	0.49	0.32	0.78
Σ	325	178			
\bar{p}	178/325 = 0.55				



$$\text{Cont Limits} = \bar{p} \pm z\sigma_p = \bar{p} \pm z \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

In each case (for each class), the sample size n varies. ∴ the SD $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, and hence **LCL & UCL** for each case vary, and form wiggly lines

c-Chart ... 1/2

Example 8:

100 workers are transported to the project site daily in buses. The number of workers missing the buses in the last 10 days has been observed to be 9,8,5,7,9,8,9,4,9,12. Develop a 3-Sigma c-Chart for the defaulting workers

Day	Number of Workers i.e. Sample Size (n)	Those who missed the buses i.e. Defaults/ Defects (d)
1	100	9
2	100	8
3	100	5
4	100	7
5	100	9
6	100	8
7	100	9
8	100	4
9	100	9
10	100	12

$$\text{Cont Limits} = \bar{c} \pm z\sqrt{\bar{c}}$$

where

\bar{c} = CL & mean number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

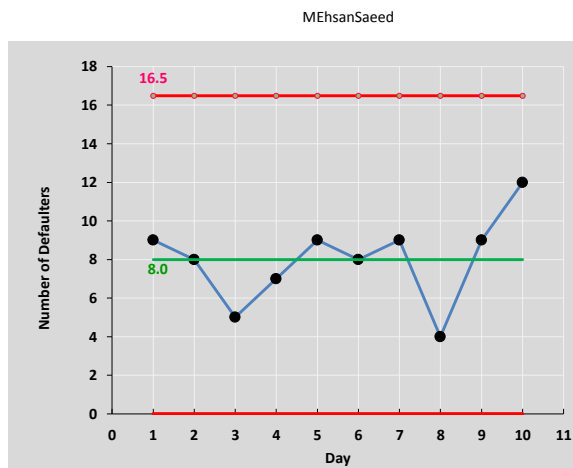
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40

c-Chart ... 2/2

Day	Defaults/ Defects	Prop Def	LCL	UCL
1	9	Not Reqd	-0.5	16.5
2	8		-0.5	16.5
3	5		-0.5	16.5
4	7		-0.5	16.5
5	9		-0.5	16.5
6	8		-0.5	16.5
7	9		-0.5	16.5
8	4		-0.5	16.5
9	9		-0.5	16.5
10	12		-0.5	16.5
Σ	80			

$$\bar{c} = 80/10 = 8.0 \text{ per day}$$



$$\text{Cont Limits} = \bar{c} \pm z\sqrt{\bar{c}}$$

Here, the SD $\sqrt{\bar{c}}$ is constant \therefore LCL & UCL for all observations are also constant, giving straight lines $\bar{c} = \frac{80}{10} = 8.0$, $z=3$

$$\text{UCL, LCL} = 8.0 + 3\sqrt{8.0} = 16.5 \rightarrow 17, -0.5 \rightarrow 0$$

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41

u-Chart ... 1/2

Example 9:

A construction company transports its workers to project site in buses. The number of workers on the company's register over the last 10 days, and those who missed the buses are as tabulated. Develop a 3-Sigma u-Chart for the defaulting workers

Day	Number of Workers i.e. Sample Size (n)	Those who missed the buses i.e. Defaults/ Defects (d)
1	98	9
2	100	8
3	100	5
4	102	7
5	100	9
6	99	8
7	99	9
8	100	4
9	100	9
10	102	12

$$\text{Cont Limits} = \bar{u} \pm z \sqrt{\frac{\bar{u}}{n}}$$

where

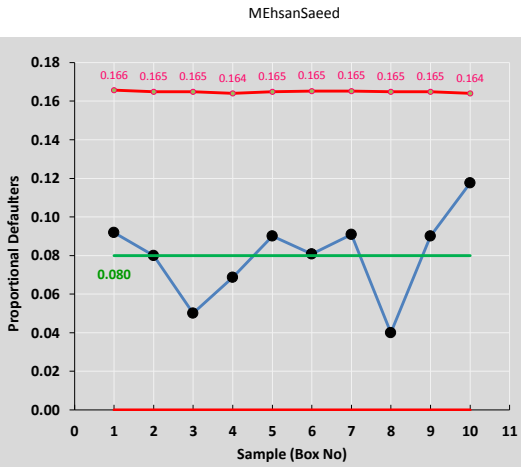
\bar{u} = CL & mean proportional number of defects

z = Quality standard, eg 3 sigma or more, or degree of confidence

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u-Chart ... 2/2

Day	Sample Size (n)	Defaults/ Defects (d)	Prop Defaults (d/n)	LCL	UCL
1	98	9	0.092	-0.006	0.166
2	100	8	0.080	-0.005	0.165
3	100	5	0.050	-0.005	0.165
4	102	7	0.069	-0.004	0.164
5	100	9	0.090	-0.005	0.165
6	99	8	0.081	-0.005	0.165
7	99	9	0.091	-0.005	0.165
8	100	4	0.040	-0.005	0.165
9	100	9	0.090	-0.005	0.165
10	102	12	0.118	-0.004	0.164
Σ	1,000	80			
\bar{u}	80/1,000 = 0.08				



$$\text{Cont Limits} = \bar{u} \pm z \sqrt{\frac{\bar{u}}{n}}$$

In each case (for each day), the sample size n varies. \therefore the standard deviation $\sqrt{\frac{\bar{u}}{n}}$, and hence **LCL & UCL** for each case vary, and form wiggly lines

np-Chart ... 1/2

Example 10:

On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 20 switches each from the 10 boxes. He finds 3, 3, 4, 2, 1, 3, 2, 3, 2 & 1 switches defective, in the ten boxes. Develop a 3-sigma np-Chart for the sampling done.

$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1 - \bar{p})}$$

where

$$\bar{p} = \text{CL \& mean of sample proportion defectives} = \frac{\text{total defectives}}{\text{total observations}}$$

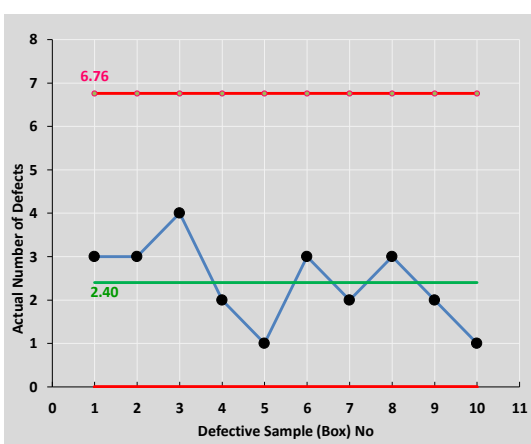
Z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$$\sigma_{np} = \text{Std Dev of the Average Proportion Defective} = \sqrt{n\bar{p}(1 - \bar{p})}$$

n = mean sample size

np-Chart ... 2/2

Def Box No	Sample Size (n)	Defects (d)	(Prop defects (d/n))	LCL	UCL	
1	20	3		-1.96	6.76	
2	20	3		-1.96	6.76	
3	20	4		-1.96	6.76	
4	20	2		-1.96	6.76	
5	20	1		-1.96	6.76	
6	20	3	Not Reqd	-1.96	6.76	
7	20	2		-1.96	6.76	
8	20	3		-1.96	6.76	
9	20	2		-1.96	6.76	
10	20	1		-1.96	6.76	
Σ		24				
$n\bar{p}$	24/10 = 2.4					
\bar{p}	2.4/20 = 0.12					



$$\text{Cont Limits} = n\bar{p} \pm z\sigma_{np} = n\bar{p} \pm z\sqrt{n\bar{p}(1 - \bar{p})}$$

In each case (for each box), the sample size **n** is the same. ∴ the SD $\sqrt{n\bar{p}(1 - \bar{p})}$, and hence **LCL & UCL** for each case are also constant, giving straight lines

$$\text{UCL, LCL} = 2.4 \pm 3\sqrt{2.4(1 - 0.12)} = 6.76 \rightarrow 7, -1.96 \rightarrow 0$$

p-Chart ... 1/2

Example 11:

On a large construction project, 10 boxes of electrical switches have arrived. Each box has 1,000 switches. Randomly, the procurement manager picks up 22, 20, 18, 18, 18, 20, 15, 18, 18 & 20 switches from the 10 boxes. He finds 3, 2, 1, 2, 1, 3, 3, 2, 1 & 1 switches defective, respectively, in the ten boxes. Develop a 3-sigma p-Chart for the defective boxes.

$$\text{Cont Limits} = \bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where

$$\bar{p} = \text{CL \& mean of sample proportion defectives} = \frac{\text{total defectives}}{\text{total observations}}$$

Z = standard deviation of sample means, eg 3 sigma or more, or degree of confidence

$$\sigma_p = \text{Std Dev of the Average Proportion Defective} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

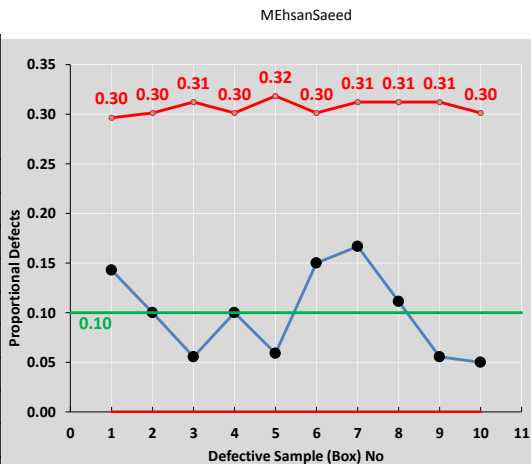
n = mean sample size

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46

p-Chart ... 2/2

Def Box No	Sample Size (n)	Defects (d)	Prop Def-ects (d/n)	LCL	UCL
1	21	3	0.14	-0.10	0.30
2	20	2	0.10	-0.10	0.30
3	18	1	0.06	-0.11	0.31
4	20	2	0.10	-0.10	0.30
5	17	1	0.06	-0.12	0.32
6	20	3	0.15	-0.10	0.30
7	18	3	0.17	-0.11	0.31
8	18	2	0.11	-0.11	0.31
9	18	1	0.06	-0.11	0.31
10	20	1	0.05	-0.10	0.30
Σ	190	19			
\bar{p}	19/190 = 0.100				



$$\text{Cont Limits} = \bar{p} \pm z\sigma_p = \bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

In each case (for each box), the sample size **n** varies. ∴ the SD $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, and hence **LCL & UCL** for each case vary, and form wiggly lines

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47